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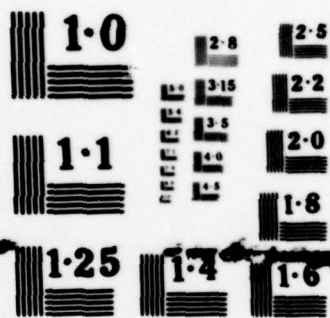UNCLASSIFIED   NL

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NONLINEAR DEVELOPMENT OF THE NEGATIVE-MASS INSTABILITY AND ITS

EFFECT ON INTENSE MICROWAVE GENERATION

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ABSTRACT

The nonlinear development of the negative-mass instability
is investigated for a thin E-layer ($\Delta << R_0$) located inside a
cylindrical waveguide. A quasilinear analysis shows that
the instability saturates by the development of an energy
spread and a corresponding increase in thickness Δ of the E-
layer. The efficiency of microwave generation by this
instability is also calculated.

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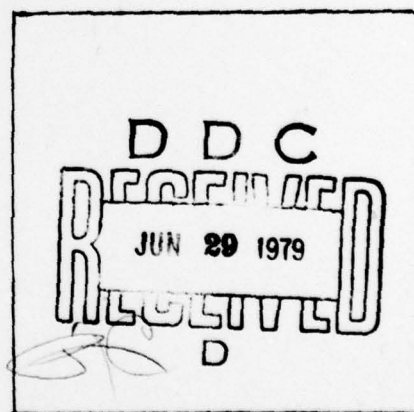
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1. INTRODUCTION AND ASSUMPTIONS

The negative-mass instability[1-9] has received considerable attention as a mechanism for intense microwave generation.[2-5] For the most part, however, analyses of this instability have been limited to the linear regime. The purpose of this paper is to investigate the nonlinear development of the negative-mass instability, with particular emphasis on the implications for intense microwave generation. The analysis is carried out for a thin ($\Delta << R_0$), relativistic nonneutral E-layer aligned parallel to a uniform axial magnetic field $B_0 \hat{e}_z$ and located inside a grounded cylindrical waveguide. The nonlinear consequences of the negative-mass instability are investigated within the framework of a simple quasilinear model that makes use of total energy balance for the system. One of the most important features of the analysis is that the instability saturates by the development of an energy spread and a corresponding increase in thickness Δ of the E-layer. The corresponding efficiency of microwave generation is also calculated.

As illustrated in Fig. 1, the present analysis is carried out for an unneutralized E-layer aligned parallel to a uniform external magnetic field $B_0 \hat{e}_z$ and located inside a cylindrical conducting waveguide with radius R_c. Cylindrical polar coordinates (r, θ, z) are introduced, and the main assumptions are enumerated below.

(a) The E-layer is thin, i.e., $\Delta/R_0 << 1$, where $\Delta = R_2 - R_1$ is the radial thickness and $R_0 = (R_1 + R_2)/2$ is the mean radius of the E-layer.

(b) The electron motion is ultrarelativistic ($\gamma_0 >> 1$) and the mean equilibrium motion of the E-layer is in the azimuthal direction (i.e., $v_z^0 = 0$, where v_z^0 is the mean axial velocity of an electron fluid element).

(c) It is further assumed that $\nu/\gamma_0 \ll 1$, where $\nu = N_e e^2/mc^2$ is Budker's parameter, N_e is the number of electrons per unit axial length of the E-layer, c is the speed of light in vacuo, $\gamma_0 mc^2$ is the azimuthal electron energy at equilibrium radius R_0, and $-e$ and m are the charge and rest mass, respectively, of the electron.

2. THEORETICAL MODEL

The equilibrium and negative-mass stability properties have been investigated in Refs. 7-9 for the choice of a sharp-boundary equilibrium in which the electrons have a rectangular density profile, i.e., $n_e^0(r) = N_e/(2\pi R_0\Delta) = $const., for $R_1 < r < R_2$, and $n_e^0(r) = 0$, otherwise. Here R_2 is the outer radius of the E-layer and $R_1 = R_2 - \Delta$ is the inner radius. For a thin E-layer, the resulting dispersion relation for the negative-mass instability can be expressed as[7-9]

$$(\omega - \ell\omega_c)^2 = -\frac{c^2}{R_0^2}\left\{\frac{2\nu}{\gamma_0}\frac{g}{\ell} - \left(\frac{\ell\Delta\omega_c}{2c}\right)^2\right\}, \tag{1}$$

where ω is the complex eigenfrequency, $\omega_c = eB_0/\gamma_0 mc$ is the electron cyclotron frequency, ℓ is the azimuthal harmonic number, and use has been made of the cold-fluid limit of the kinetic dispersion relation[8,9] for the negative-mass instability. In Eq. (1), the geometric factor g is defined by $g = \ell^2/(b_- + b_+) - k^2R_0^2/(d_- + d_+)$, and

$$b_+ = -\frac{\ell J_\ell(pR_2)N_\ell'(pR_c) - J_\ell'(pR_c)N_\ell(pR_2)}{R_2 p J_\ell'(pR_2)N_\ell'(pR_c) - J_\ell'(pR_c)N_\ell'(pR_2)},$$

$$d_+ = -\frac{R_2 p J_\ell'(pR_2)N_\ell(pR_c) - J_\ell(pR_c)N_\ell'(pR_2)}{\ell J_\ell(pR_2)N_\ell(pR_c) - J_\ell(pR_c)N_\ell(pR_2)}, \tag{2}$$

$$b_- = \ell J_\ell(pR_1)/[R_1 p J_\ell'(pR_1)], \quad d_- = 1/b_-,$$

where $J_\ell(x)$ and $N_\ell(x)$ are Bessel functions of the first and second kind, respectively, the prime (') denotes $(1/p)(d/dr)$, and p is defined

by $p=(\omega^2/c^2-k^2)^{1/2}$, where k is the axial wavenumber.

The nonlinear consequences of the negative-mass instability can be investigated within the framework of the conservation equation for net energy balance.[10] We Fourier decompose perturbed quantities according to $\delta A(\underset{\sim}{x},t)=\sum\limits_{\ell,k}\delta A_{\ell,k}(r,t)\exp(i\ell\theta+ikz)$, where $k=2\pi q/L$ and q is an integer. Energy conservation can then be expressed as

$$\frac{\partial}{\partial t}\sum_{\ell,k}2\pi\int_0^{R_c}drr\epsilon_{\ell,k}(r,t)\ +\frac{\partial}{\partial t}\ K=0,\tag{3}$$

where $\epsilon_{\ell,k}=|\delta\underset{\sim}{E}_{\ell,k}(r,t)|^2/8\pi+|\delta\underset{\sim}{B}_{\ell,k}(r,t)|^2/8\pi$ is the perturbed field energy density, and

$$K=\frac{1}{L}\int_{-L/2}^{L/2}dz\int_0^{2\pi}d\theta\int_0^{R_c}drr\int d^3p(\gamma-1)mc^2f_e(\underset{\sim}{x},\underset{\sim}{p},t)$$

is the average electron kinetic energy per unit length. Here L is an effective periodicity length. Within the context of a simple quasilinear model, the spectral energy density evolves according to[11]

$$\frac{\partial}{\partial t}\epsilon_{\ell,k}(r,t)=2\omega_i^{\ell,k}\epsilon_{\ell,k}(r,t)\ ,\tag{4}$$

where the growth rate $\omega_i^{\ell,k}=\mathrm{Im}\omega$ is determined adiabatically in terms of other system parameters from the linear dispersion relation in Eq. (1). Moreover, for a thin E-layer with $V_z^0=0$ and near-laminar flow in the azimuthal direction, we approximate the electron kinetic energy per unit length by[10]

$$K=2\pi\int_0^{R_c}drrn_e^0(r,t)(\gamma-1)mc^2\ ,\tag{5}$$

where $n_e^0(r,t)=\int_0^{2\pi}\frac{d\theta}{2\pi}\int_{-L/2}^{L/2}\frac{dz}{L}n_e(\underset{\sim}{x},t)$ is the average electron density, and $\gamma mc^2\approx(1+r^2\hat\omega_c^2/c^2)^{1/2}mc^2$ is the azimuthal energy associated with the average electron motion $(V_\theta^0\approx r\hat\omega_c/\gamma)$. Here $\hat\omega_c=eB_0/mc$ is the nonrelativistic

electron cyclotron frequency.

Substituting Eqs. (4) and (5) into Eq. (3), and assuming that the electron density profile maintains a rectangular shape with variable thickness $\Delta(t)$, we obtain

$$\frac{1}{3} N_e mc^2 F(\Delta) \frac{d\Delta}{dt} = \sum_{\ell,k} \omega_i^{\ell,k} \bar{W}_{\ell,k}(0) \exp\left\{2\int_0^t dt' \omega_i^{\ell,k}(t')\right\} , \quad (6)$$

where $\bar{W}_{\ell,k}(0) \equiv 2\pi \int_0^{R_c} dr r \varepsilon_{\ell,k}(r,t=0)$ characterizes the energy of the initial field perturbations, N_e is the number of electrons per unit axial length, and $F(\Delta) > 0$ is defined by

$$F(\Delta) = \left\{ \frac{2x_1 [(1+x_2^2)^{3/2} - (1+x_1^2)^{3/2}]}{(x_2^2 - x_1^2)^2} - \frac{3x_1(1+x_1^2)^{1/2}}{x_2^2 - x_1^2} \right\} \frac{\hat{\omega}_c}{c} , \quad (7)$$

with $x_2 = \hat{\omega}_c R_2/c$ and $x_1 = \hat{\omega}_c (R_2 - \Delta)/c$. Consistent with experimental observations,[3] in obtaining the left-hand side of Eq. (6) we have approximated $dR_2/dt \approx 0$ but retained terms proportional to $d\Delta/dt$.

3. NONLINEAR EVOLUTION

For $g > 0$, it is evident from Eq. (1) that the linear growth rate $\omega_i^{\ell,k}$ vanishes once the layer thickness has increased to the critical value

$$\Delta_f = \frac{2c}{\ell\omega_c} \left(\frac{2\nu g}{\ell\gamma_0}\right)^{1/2} . \quad (8)$$

Therefore, making use of Eq. (6), the quasilinear evolution of the system can be summarized as follows. At $t=0$, the initial thickness of the E-layer is $\Delta_i = \Delta(t=0)$. As the field perturbations grow, $\Delta(t)$ continues to increase [$d\Delta/dt > 0$ since $F(\Delta) > 0$] until $\Delta(t \to \infty) = \Delta_f$ and the instability ceases. (Here we assume that a single ℓ value is excited.) Concomitant with the increase in E-layer thickness is an increase in azimuthal energy spread of the electrons composing the layer.[8,9]

The efficiency η of microwave generation is defined as the ratio of electromagnetic field energy generated by the instability to the initial kinetic energy of the E-layer. In the ultrarelativistic limit with $\gamma_0 \gg 1$, $x_2^2 \gg 1$, $x_1^2 \gg 1$ and $\Delta/R_0 \ll 1$, some straightforward algebra that makes use of Eqs. (6) and (7) shows that η can be approximated by

$$\eta = \frac{1}{2R_0} (\Delta_f - \Delta_i) \ . \tag{9}$$

For future reference, we also introduce the average power per unit axial length associated with the microwave production, i.e.,

$$\bar{P}_{\ell,k} = \frac{2\pi}{T} \int_0^{R_c} r\,dr [\epsilon_{\ell,k}(r,T) - \epsilon_{\ell,k}(r,0)] \tag{10}$$

where T is the duration of the radiation.

Equations (1) and (6) have been solved numerically for the self-consistent nonlinear evolution of the E-layer thickness $\Delta(t)$. For present purposes, we consider transverse electric (TE mode) perturbations with $k^2 R_0^2 = \ell^2 - \alpha_{\ell n}^2 (R_0^2/R_c^2)$ and $J_\ell'(\alpha_{\ell n}) = 0$. Typical results are summarized in Figs. 2 and 3 for $R_c/R_0 = 1.5$ and $\gamma_0 = 4.5$. In Fig. 2, we plot normalized E-layer thickness $\Delta(t)/R_0$ versus $\omega_c t$ for several different values of Budker's parameter $\nu = N_e e^2/mc^2$. Evidently, the time-asymptotic thickness of the E-layer (Δ_f) is a slowly increasing function of ν [see also Eq. (8)]. On the other hand, the time scale required for saturation decreases rapidly with increasing ν, which is a consequence of the fact that the growth rate $\omega_i^{\ell,k}$ increases with ν during the initial stages of instability when $\Delta \ll \Delta_f$ [Eq. (1)].

We note from Eqs. (8) and (9) that the efficiency η of microwave generation is also an increasing function of Budker's parameter ν. To illustrate this point, the normalized average microwave power per

unit length $\bar{P}/\omega_c(mc^2/e)^2$ is plotted versus ν in Fig. 3 for R_c/R_0=1.5, γ_0=4.5, and several different waveguide modes (ℓ,n). As an example, for the TE_{31} mode, the average microwave power per unit length $\bar{P}$ increases five-fold when ν is increased from 0.07 to 0.15. Also shown in Fig. 3 is a plot of efficiency η versus ν.

4. CONCLUSIONS

Several important conclusions follow from the present analysis. For example, the time evolution of the E-layer thickness exhibits a very sensitive dependence on Budker's parameter ν [Fig. 2]. Moreover, the average power and efficiency of microwave generation are enhanced considerably by an increase in electron density [Fig. 3]. Introducing an axial energy spread into the present analysis reduces the growth rate $\omega_i^{\ell,k}$ substantially,[5,9] thereby slowing the rate at which the E-layer thickness increases. This behavior is consistent with recent experimental observations by Destler et al. (cf., Figs. 8 and 11 in Ref. 3).

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Fig. 1   Equilibrium configuration and electron density profile.

Fig. 2   Plot of $\Delta(t)/R_0$ versus $\omega_c t$ for $R_c/R_0 = 1.5$, $\gamma_0 = 4.5$, $\Delta_1/R_0 = 2 \times 10^{-2}$, $\bar{W}_{\ell,k}(0)/(N_e mc^2) = 8 \times 10^{-3}$, and several different values of Budker's parameter ν.

Fig. 3   Plot of normalized microwave power per unit length $\bar{P}/\omega_c (mc^2/e)^2$ versus ν [Eqs. (10) and (6)] for $R_c/R_0 = 1.5$, $\gamma_0 = 4.5$, $\Delta_1/R_0 = 2 \times 10^{-2}$, $\bar{W}_{\ell k}(0) = 8 \times 10^{-3} N_e mc^2$, and several different modes (ℓ, n). Also shown is a plot of efficiency η versus ν.
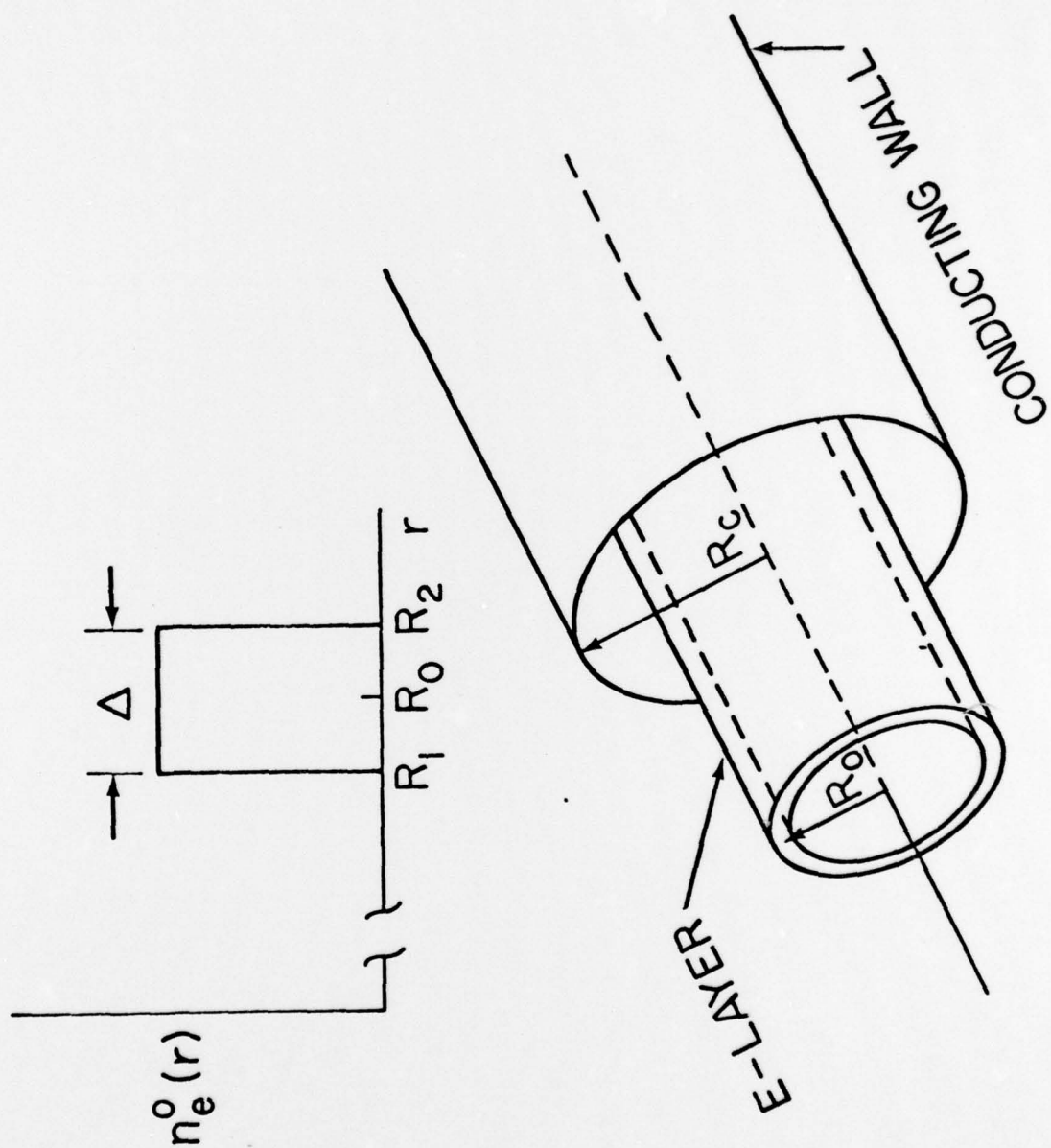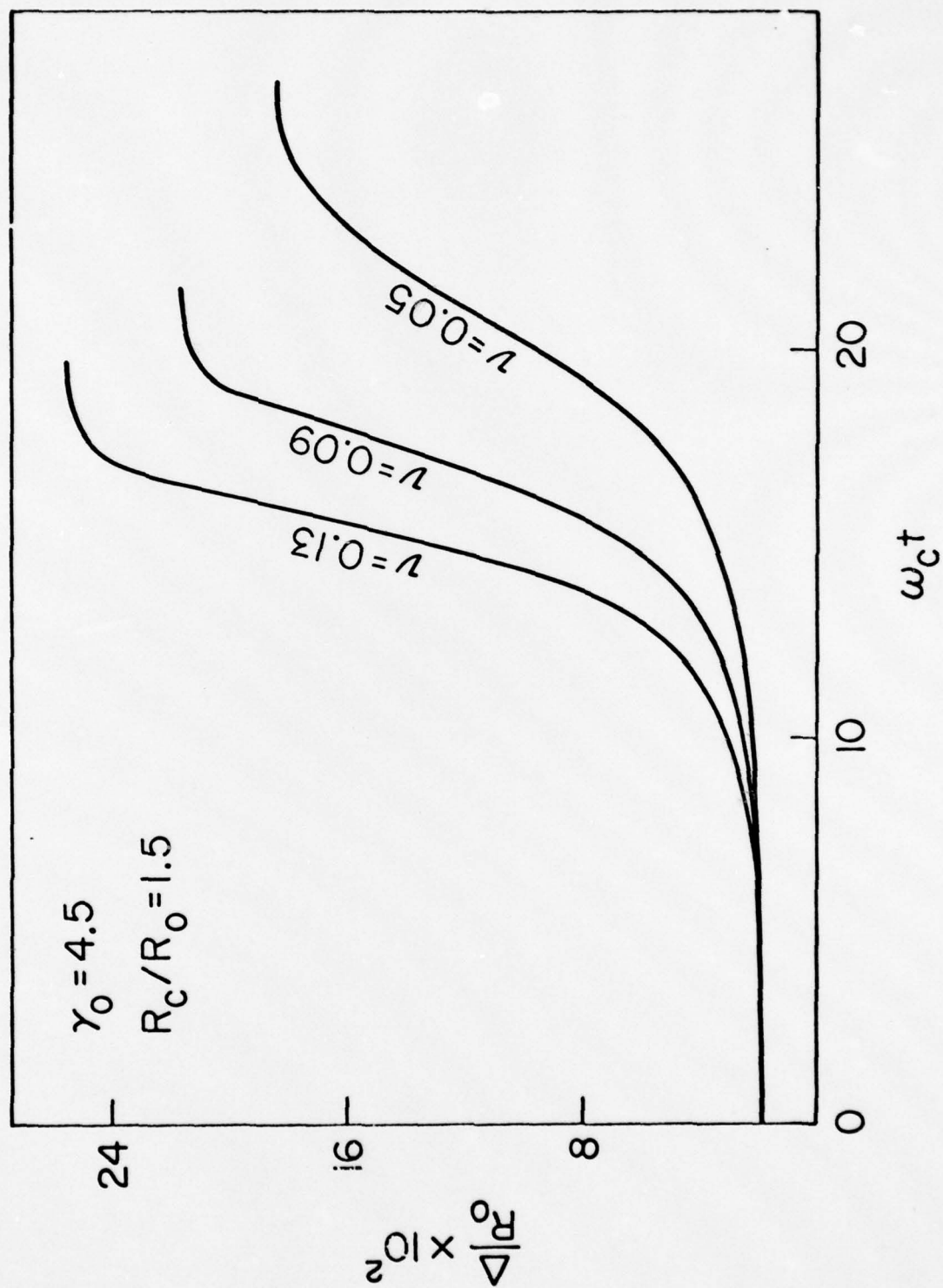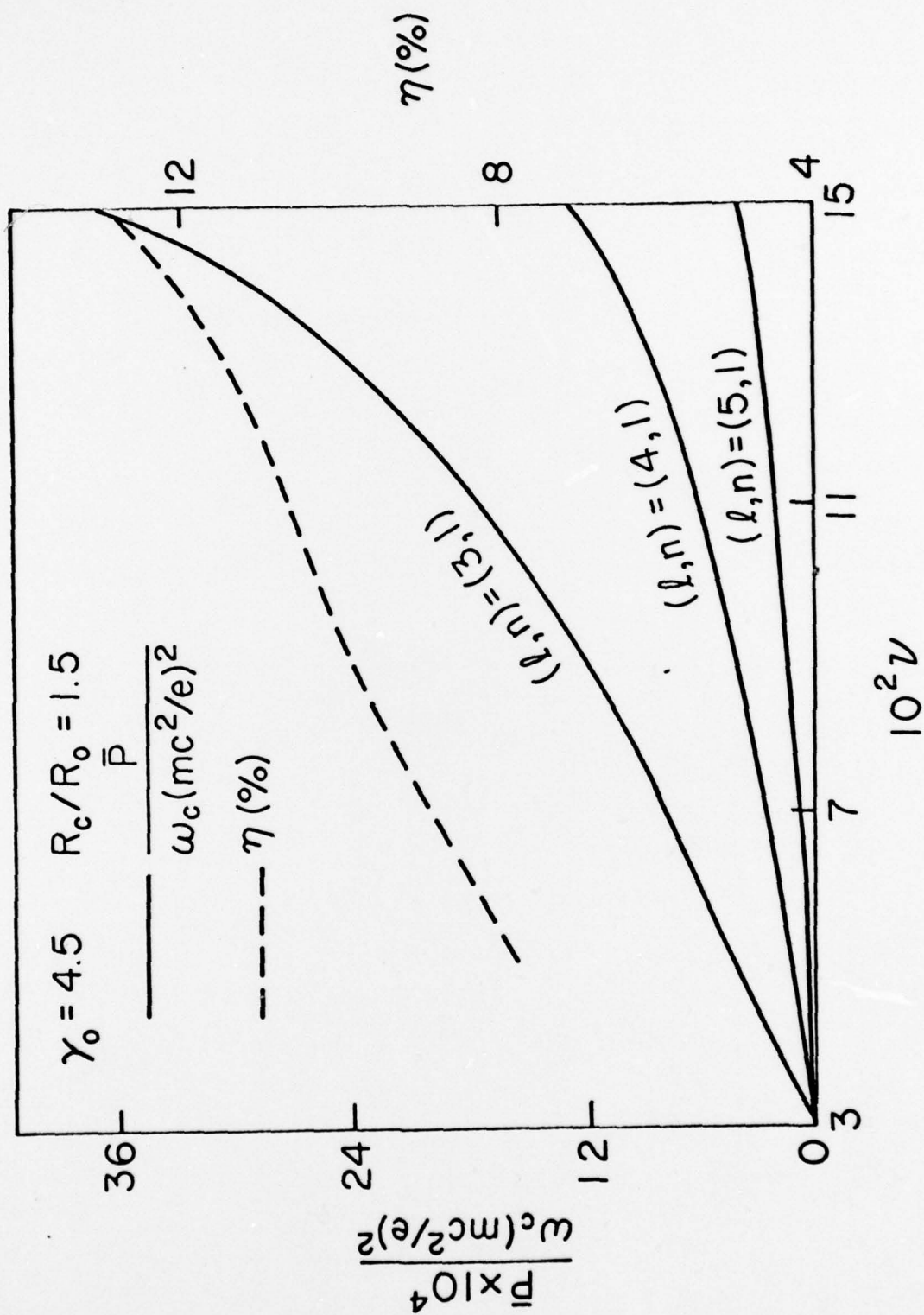
CONDUCTING WALL

R_c

R_0

E-LAYER

$n_e^0(r)$

Δ

R_1 R_0 R_2 r

Fig. 1

Fig. 2

Fig. 3